SCIENCE CHINA
Information Sciences

• LETTER •

September 2019, Vol. 62 199203:1–199203:3
https://doi.org/10.1007/s11432-018-9677-9

Tracking control and parameter identification with quantized ARMAX systems

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Received 17 October 2018/Accepted 26 November 2018/Published online 29 July 2019

Citation Jing L D, Zhang J F. Tracking control and parameter identification with quantized ARMAX systems. Sci China Inf Sci, 2019, 62(9): 199203, https://doi.org/10.1007/s11432-018-9677-9

Dear editor,

After decades of development, system modeling and control theory have developed thoroughly. Various identification, estimation and control methods have been given [1]. However, most of these results are based on accurate measurements of the system inputs and outputs. But now, quantized data are more and more popular, and how to use quantized data to model and control the systems is of great importance and involved in many challenging problems. Ref. [2] studied the quantized partial-state feedback stabilization of a class of nonlinear cascaded systems and gave a recursive design method for quantized stabilization. Ref. [3] used quantized data to design feedback stabilization control. Ref. [4] studied the parameter identification of set value systems. Ref. [5] considered the adaptive control of first-order systems. Ref. [6] solved the adaptive tracking control of linear systems with binary-valued observations and periodic target.

Unlike precise measurements, when using quantized data we have to consider the effects of quantization error, which cannot be simply assumed as zero mean white noise and is different from white noises, since quantization error depends on both system inputs and system noises.

This study consists of two parts. In the first part, we use the quantized output to design a tracking control and establish the relationship between the tracking error and the quantization error for a class of ARMAX systems with correlated noises. In the second part, we give a parameter identification method for a class of linear time-invariant systems with quantized outputs but without system noises, and study the influence of the quantization error on the parameter estimation error. Compared with [3–6], the model is much more general. For instance, the system here is not assumed to be of first-order or open-loop stable, and is allowed to be with autoregressive terms.

Tracking control of quantized ARMAX systems. Consider the following ARMAX systems:

$$A(z)y(k) = B(z)u(k-d) + C(z)w(k),\ d \geqslant 1, \quad (1)$$

where $y(k)$, $u(k)$ and $w(k)$ are the m-, l- and m-dimensional output, input and noise, and

$$A(z) = I + A_1 z + \cdots + A_p z^p, \quad (2)$$
$$B(z) = B_1 + B_2 z + \cdots + B_q z^{q-1}, \quad (3)$$
$$C(z) = I + C_1 z + \cdots + C_r z^r \quad (4)$$

are known and with the shift-back operator z.

The orders p, q, r and the time-delay d are assumed known.

For simplicity, we suppose $y(k) = u(k) = \omega(k) = 0$, for any $k < 0$.

The task here is to design tracking control only using inputs and quantized outputs.

For a given constant $\varepsilon > 0$ and any $k = 1, 2, \ldots$, the quantizer we use here is of the following uni-

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form one:

$$s_i(k)=\begin{cases}\vdots\\ -\varepsilon, & y_i(k)\in\left[-\dfrac{3\varepsilon}{2},-\dfrac{\varepsilon}{2}\right),\\ 0, & y_i(k)\in\left[-\dfrac{\varepsilon}{2},\dfrac{\varepsilon}{2}\right),\\ \varepsilon, & y_i(k)\in\left[\dfrac{\varepsilon}{2},\dfrac{3\varepsilon}{2}\right),\\ \vdots\end{cases}\tag{5}$$

where $y_i(k)$ is the ith component of output $y(k)$, for $i=1,2,\ldots,m$.

$$s(k)=[s_1(k),\ldots,s_m(k)]^{\mathrm{T}}.\tag{6}$$

Lemma 1 ([7]). The Diophantine equation

$$\det(C(z))I=F(z)\mathrm{adj}(C(z))A(z)+z^dG(z)\tag{7}$$

has the unique solution $F(z)$ and $G(z)$, where $F(z)=F_0+F_1z+\cdots+F_{d-1}z^{d-1}$ with $F_0=I$.

Assumption 1. $B(z)$ and $C(z)$ are stable, i.e., $\det(B(z))\neq 0$ and $\det(C(z))\neq 0$, for any $|z|\leqslant 1$, B_1 is non-degenerate.

Assumption 2. $y^*(k)$ is the tracking signal and satisfies $\limsup_{t\to\infty}\frac{1}{t}\sum_{k=0}^{t}||y^*(k)||^2<\infty$.

Assumption 3. $\{w(k),\mathcal{F}_k\}$ is a martingale difference sequence ($\mathcal{F}_k$ is non-descending subalgebra) and $\sup_k\mathrm{E}[||w_{k+1}||^2|\mathcal{F}_k]<\gamma<\infty$, almost sure, $\lim_{k\to\infty}\frac{1}{k}\sum_{i=0}^{k}||w(i)||^2=R>0$.

Denote $F(z)\mathrm{adj}(C(z))B(z)=D_0+D_1z+\cdots+D_{p_1}z^{p_1}$. Then, $D_0=B_1$ is non-degenerate.

From [7] and (5), the tracking control can be intuitively chosen to satisfy

$$\begin{aligned}&F(z)\mathrm{adj}(C(z))B(z)u(k)\\&\quad=\det(C(z))y^*(k+d)-G(z)s(k),\end{aligned}\tag{8}$$

or equivalently,

$$\begin{aligned}u(k)=&D_0^{-1}(\det(C(z))y^*(k+d)\\&-G(z)s(k)-\sum_{i=1}^{p_1}D_iu(k-i)).\end{aligned}$$

Theorem 1. For the system (1), if Assumptions 1 and 3 are satisfied, then

$$\begin{aligned}&\limsup_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n}||y(k)-y^*(k)||^2\\&=\mathrm{tr}\sum_{j=0}^{d-1}F_jRF_j^{\mathrm{T}}+O(\varepsilon).\end{aligned}\tag{9}$$

Proof. See Appendix A.

Theorem 2. For the system (1), with the control (8), if Assumptions 1–3 are satisfied, the closed-loop system is stable in the sense that

$$\limsup_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n}(||y(k)||^2+||u(k)||^2)<\infty.\tag{10}$$

Proof. See Appendix B.

Remark 1. Here we consider only the uniform quantizers. In this case, no matter what the system inputs and outputs are and whether or not the system involved is stable, the quantization error is always uniformly bounded, which makes the stabilization and analysis of the closes-loop systems easier. However, when other type of quantizers, for instance, logarithmic quantizers [8], are used, the quantization error may be unbounded, which makes the control synthesis much more difficult.

Parameter identification of quantized DARMA systems. Consider the following DARMA systems:

$$A(z)y(k)=z^dB(z)u(k),\ d\geqslant 1,\tag{11}$$

where $y(k)$, $u(k)$ are the m-, l- dimensional output, input. $A(z)$ and $B(z)$ are of the form of (2) and (3), but with unknown coefficient matrices. The orders p and q and the time-delay d are assumed known, z is the shift-back operator.

The purpose of the following part is to estimate the unknown coefficient matrices of $A(z)$ and $B(z)$. To do so, we need the following assumptions.

Assumption 4. $A(z)$ and $B(z)$ are left-coprime and A_p is of full rank.

Assumption 5. There are constants $V>0$ and $\delta>0$ such that $||u(i)||\leqslant V$ and

$$\sum_{i=k+1}^{k+h}U_iU_i'\geqslant\delta I,\quad k\geqslant 0,$$

where $h\geqslant(mp+q)l$, and

$$U_i=\left[u^{\mathrm{T}}(i),\ldots,u^{\mathrm{T}}(i-mp-q+1)\right]^{\mathrm{T}}.\tag{12}$$

Lemma 2 ([9]). Let

$$\begin{aligned}H_x(z)=&x_1^{\mathrm{T}}\mathrm{adj}(A(z))B(z)z^d+\cdots\\&+x_p^{\mathrm{T}}\mathrm{adj}(A(z))B(z)z^{p+d-1}\\&+x_{p+1}^{\mathrm{T}}z^{d-1}\det(A(z))+\cdots\\&+x_{p+q}^{\mathrm{T}}z^{q+d-2}\det(A(z))\\=&\sum_{i=0}^{mp+q-1}g_i^{\mathrm{T}}(x)z^{i+d-1},\end{aligned}$$

where $x\in\mathbb{R}^{mp+lq}$ is in the vector-component form $x=[x_1^{\mathrm{T}},x_2^{\mathrm{T}},\ldots,x_{p+q}^{\mathrm{T}}]^{\mathrm{T}}$ with $x_i\in\mathbb{R}^m$, $x_j\in\mathbb{R}^l$, $1\leqslant i\leqslant p$, $p+1\leqslant j\leqslant p+q$. If Assumptions 4 and 5 are satisfied, then $\min_{||x||=1}||g(x)||^2>0$, where $g(x)=[g_0^{\mathrm{T}}(x),g_1^{\mathrm{T}}(x),\ldots,g_{mp+q-1}^{\mathrm{T}}(x)]^{\mathrm{T}}$.

Lemma 3. Let $H'_x(z)=\sum_{i=1}^{p}x_i^{\mathrm{T}}\mathrm{adj}(A(z))z^{i-1}$. If Assumptions 4 and 5 are satisfied, there exists a constant $c_0>0$ such that $|H_x(z)u(i)H'_x(z)\epsilon(i)|\leqslant\frac{c_0}{3}\varepsilon$, for any $x\in\mathbb{R}^{mp+lq}$, $||x||=1$.

Proof. See Appendix C.

Let $\theta=[-A_1,\ldots,-A_p,B_1,\ldots,B_q]^{\mathrm{T}}$. We now estimate the unknown parameter θ by the following projection algorithm [9]:

$$\theta_{n+1}=\theta_n+\frac{\varphi_n}{1+||\varphi_n||^2}(s^{\mathrm{T}}(n+1)-\varphi_n^{\mathrm{T}}\theta_n), \quad (13)$$

$$\begin{aligned}\varphi_n^{\mathrm{T}}=&[s^{\mathrm{T}}(n),\ldots,s^{\mathrm{T}}(n-p+1),\\&u^{\mathrm{T}}(n-d+1),\ldots,u^{\mathrm{T}}(n-q-d+2)]\end{aligned}$$

with arbitrary initial values θ_0 and φ_0.

Set

$$\begin{cases}\tilde{\theta}_n=\theta-\theta_n,\\ \Psi(n+1,i)=\left(I-\dfrac{\varphi_n\varphi_n^{\mathrm{T}}}{1+||\varphi_n||^2}\right)\Psi(n,i),\\ \Psi(i,i)=I.\end{cases} \quad (14)$$

Lemma 4. Suppose Assumption 4 holds and Assumption 5 is satisfied for $\delta=\frac{hc_0\varepsilon}{\min_{||x||=1}||g(x)||^2}$. Then, there is a constant $c>0$ such that

$$\begin{aligned}&\lambda_{\min}\left(\sum_{i=k+1}^{k+mp+h}\varphi_i\varphi_i^{\mathrm{T}}\right)\\&\geqslant\frac{c}{h}\lambda_{\min}\left(\sum_{i=k+mp-d+2}^{k+mp+h-d+1}U_iU_i^{\mathrm{T}}\right),\end{aligned} \quad (15)$$

for any $k\geqslant 0$, where $\lambda_{\min}(X)$ is the minimum eigenvalue of matrix X and U_i is defined by (12).

Proof. See Appendix D.

Lemma 5. See Appendix E.

For $i=0,1,2,\ldots$, let $\tau_i=i(h+mp)+1$ and $M_i=\sup_{\tau_{i-1}\leqslant j\leqslant\tau_i-1}||\varphi_j||^2+1$. Then, we have the following lemma.

Lemma 6. Suppose the conditions of Lemma 4 hold. Then, there are constants $c_1>0$ and $c_2>0$ such that $||\Psi(\tau_n,0)||\leqslant\exp(-c_1\sum_{i=1}^{n}\frac{\delta^2}{M_i^2})$, and $||\Psi(\tau_n,\tau_{n-1})||\leqslant\exp(-c_2\frac{\delta^2}{M_n^2})$.

Proof. See Appendix F.

Theorem 3. Suppose the conditions of Lemma 4 hold and there is a constant $v\in[0,\frac{1}{4})$ such that

$$||\varphi_n||=O\left(n^v\right),\quad\forall n, \quad (16)$$

then, we have $||\tilde{\theta}_n||=O\left(\varepsilon\right)$.

Proof. See Appendix G.

Remark 2. Because $\varepsilon>0$ can be chosen arbitrarily small, so are the δ in Lemmas 4 and 6, and Theorem 3. Thus, Assumption 5 can be realized by properly choosing the input $u(i)$ and ε.

Conclusion. In this study, we have studied the tracking control and parameter identification of linear time-invariant discrete-time system with quantized outputs. By using quantized output data from a uniform quantizer, we designed the tracking control law and shown the stability of the closed-loop system and the suboptimality of the tracking error. As for parameter identification, we proved the boundness of parameter estimation error by using the projection algorithm. In addition, numerical simulations have been given in Appendix H. But for other type of quantizers or nonlinear systems, parameter identification and tracking control based on quantized output data have not been solved and need further study.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 61877057, 61227902).

Supporting information Appendixes A–H. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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• **Supplementary File** •

Tracking Control and Parameter Identification with Quantized ARMAX Systems

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Appendix A Proof of Theorem 1

By (1) and (7) we have

$$\begin{aligned}
&\det(C(z))\,(y(k)-F(z)w(k))\\
=\;& F(z)\mathrm{adj}\,(C(z))\,A(z)y(k)+G(z)y(k-d)-\det(C(z))\,F(z)w(k)\\
=\;& F(z)\mathrm{adj}\,(C(z))\,B(z)u(k-d)+F(z)\det(C(z))\,w(k)+G(z)y(k-d)-\det(C(z))\,F(z)w(k)\\
=\;& G(z)y(k-d)+F(z)\mathrm{adj}\,(C(z))\,B(z)u(k-d),
\end{aligned}$$

which together with (8) leads to

$$\begin{aligned}
\det(C(z))\,(y(k)-F(z)w(k)) &= G(z)y(k-d)+\det(C(z))\,y^*(k)-G(z)s(k-d),\\
\det(C(z))\,(y(k)-y^*(k)) &= G(z)(y(k-d)-s(k-d))+\det(C(z))\,F(z)w(k).
\end{aligned}$$

Thus, by Assumptions 1 and 3 and (6) we have

$$\limsup_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n}||y(k)-y^*(k)||^2=tr\sum_{j=0}^{d-1}F_jRF_j^T+O(\varepsilon).$$

Appendix B Proof of Theorem 2

From (1) it is easy to see

$$B(z)u(k-d)=A(z)y(k)-C(z)w(k).$$

Notice that

$$\frac{1}{n}\sum_{k=0}^{n}||y(k)||^2=\frac{1}{n}\sum_{k=0}^{n}||y(k)-y^*(k)+y^*(k)||^2\leqslant\frac{2}{n}\sum_{k=0}^{n}||y(k)-y^*(k)||^2+\frac{2}{n}\sum_{k=0}^{n}||y^*(k)||^2. \tag{B1}$$

Then, by Assumption 1, there is a constant $C'>0$ such that

$$\frac{1}{n}\sum_{k=0}^{n}||u(k)||^2\leqslant\frac{C'}{n}\sum_{k=0}^{n+d}\left(||y(k)||^2+||w(k)||^2\right).$$

This together with Assumptions 2 and 3, Theorem 1 and (B1) implies (10).

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Appendix C Proof of Lemma 3

By (5), (11) can be rewritten as

$$A(z)s(k) = z^d B(z)u(k) + \epsilon(k), k \geqslant 0,$$

where $\epsilon(k) = A(z)(s(k) - y(k))$.

By (5), one can get

$$||\epsilon(k)|| \leqslant M\varepsilon, \tag{C1}$$

with $M = \frac{m}{2}\sum_{i=0}^{p}||A_i||$.

By Assumption 5, $u(i)$ is bounded. So, there exists a constant c_0 independent of ε such that

$$|H_x(z)u(i)H'_x(z)\epsilon(i)| \leqslant \frac{c_0}{3}\varepsilon, \quad x \in R^{mp+lq}, \quad ||x|| = 1.$$

Appendix D Proof of Lemma 4

Let

$$\det(A(z)) = a_0 + a_1 z + \cdots + a_{mp}z^{mp},\ a_{mp} \neq 0,$$

and

$$\psi_n = \det(A(z))\,\varphi_n. \tag{D1}$$

Then

$$\begin{aligned}\psi_n = [&\mathrm{adj}\,(A(z))\,(z^d B(z)u(n) + \epsilon(n))^T, \cdots, \\ &\mathrm{adj}\,(A(z))\,(z^{p+d-1}B(z)u(n) + \epsilon(n-p+1))^T, \\ &z^{d-1}\det(A(z))\,u^T(n), \cdots, z^{d+q-2}\det(A(z))\,u^T(n)]^T.\end{aligned} \tag{D2}$$

From (D1) we can obtain that for any $x \in R^{mp+lq}$,

$$\begin{aligned}x'\left(\sum_{i=k+mp+1}^{k+mp+h}\psi_i\psi'_i\right)x &= \sum_{i=k+mp+1}^{k+mp+h}(x'\psi_i)^2 = \sum_{i=k+mp+1}^{k+mp+h}\left(\sum_{j=0}^{mp}a_j x'\varphi_{i-j}\right)^2 \leqslant \sum_{j=0}^{mp}a_j^2\sum_{i=k+mp+1}^{k+mp+h}\sum_{j=0}^{mp}(x'\varphi_{i-j})^2 \\ &\leqslant h\sum_{j=0}^{mp}a_j^2\sum_{i=k+1}^{k+mp+h}x'\varphi_i\varphi'_i x,\end{aligned}$$

which implies

$$\lambda_{min}\left(\sum_{i=k+1}^{k+mp+h}\varphi_i\varphi'_i\right) \geqslant \frac{1}{h\sum_{j=0}^{mp}a_j^2}\lambda_{min}\left(\sum_{i=k+mp+1}^{k+mp+h}\psi_i\psi'_i\right).$$

Hence, in order to prove (15) we only need to show that

$$\lambda_{min}\left(\sum_{i=k+mp+1}^{k+mp+h}\psi_i\psi'_i\right) \geqslant c_1\lambda_{min}\left(\sum_{i=k+mp-d+2}^{k+mp+h-d+1}U_iU'_i\right), c_1 > 0.$$

Write the unit vector $x \in R^{mp+lq}$ in the vector-component form $x = \left[x_1^T, x_2^T, \cdots, x_{p+q}^T\right]^T$. Then, by (D2), Assumption 5 and $\delta = \frac{hc_0\varepsilon}{\min_{||x||=1}||g(x)||^2}$ we have

$$\begin{aligned}&x'\sum_{i=k+mp+1}^{k+mp+h}\psi_i\psi'_i x = \sum_{i=k+mp+1}^{k+mp+h}\left(H_x(z)u(i) + H'_x(z)\epsilon(i)\right)^2 \\ =&g'(x)\sum_{i=k+mp-d+2}^{k+mp+h-d+1}U_iU'_i g(x) + 2\sum_{i=k+mp+1}^{k+mp+h}H_x(z)u(i)H'_x(z)\epsilon(i) + \sum_{i=k+mp+1}^{k+mp+h}\left(H'_x(z)\epsilon(i)\right)^2 \\ \geqslant&\min_{||x||=1}||g(x)||^2\lambda_{min}\left(\sum_{i=k+mp-d+2}^{k+mp+h-d+1}U_iU'_i\right) + 2\sum_{i=k+mp+1}^{k+mp+h}H_x(z)u(i)H'_x(z)\epsilon(i) \\ \geqslant&\min_{||x||=1}||g(x)||^2\lambda_{min}\left(\sum_{i=k+mp-d+2}^{k+mp+h-d+1}U_iU'_i\right) - \frac{2h}{3}c_0\varepsilon \\ =&\min_{||x||=1}||g(x)||^2\left(\lambda_{min}\left(\sum_{i=k+mp-d+2}^{k+mp+h-d+1}U_iU'_i\right) - \frac{2\delta}{3}\right) \\ \geqslant&\frac{1}{3}\min_{||x||=1}||g(x)||^2\lambda_{min}\left(\sum_{i=k+mp-d+2}^{k+mp+h-d+1}U_iU'_i\right).\end{aligned}$$

This together with Lemma 2 gives (15).

Appendix E Lemma 5

If

$$\sum_{i=k}^{N-1} \frac{\varphi_i \varphi_i'}{1+||\varphi_i||^2} \geqslant \alpha I,$$

for some $\alpha > 0$, then we have

$$||\Psi(N,k)|| \leqslant \left[1 - \frac{\alpha^2}{4(N-k)^3}\right]^{1/2}.$$

Proof. See [1].

Appendix F Proof of Lemma 6

For the first inequality of Lemma 6

$$||\Psi(\tau_n, 0)|| \leqslant \exp\left(-c_1 \sum_{i=1}^{n} \frac{\delta^2}{M_i^2}\right),$$

please see [1].

Here we need only to show the second inequality of Lemma 6. By Lemma 4 and Assumption 5

$$\sum_{i=\tau_{n-1}}^{\tau_n - 1} \frac{\varphi_i \varphi_i'}{1+||\varphi_i||^2} \geqslant \frac{c\delta}{M_n h} I.$$

This together with Lemma 5 and the elementary inequality $1 - x \leqslant e^{-x}, \forall x \in [0,1]$ leads to

$$||\Psi(\tau_n, \tau_{n-1})|| \leqslant \left(1 - c_2' \frac{\delta^2}{M_n^2}\right)^{\frac{1}{2}},$$

where $c_2' > 0$ is a constant.

Let $c_2 = \frac{1}{2} c_2'$. Then, we can get Lemma 6.

Appendix G Proof of Theorem 3

By (14) we have

$$\begin{aligned}
\tilde{\theta}_{n+1} =& \theta - \theta_{n+1} \\
=& \theta - \theta_n - \frac{\varphi_n}{1+||\varphi_n||^2}\left(s_{n+1} - \varphi_n' \theta_n\right) \\
=& \tilde{\theta}_n - \frac{\varphi_n}{1+||\varphi_n||^2}\left(\varphi_n' \tilde{\theta}_n + \epsilon(n+1)\right) \\
=& \left(I - \frac{\varphi_n \varphi_n'}{1+||\varphi_n||^2}\right)\tilde{\theta}_n - \frac{\varphi_n}{1+||\varphi_n||^2}\epsilon(n+1) \\
=& \cdots \\
=& \Psi(n+1, 0)\tilde{\theta}_0 - \frac{\varphi_n}{1+||\varphi_n||^2}\epsilon(n+1) - \cdots \\
& - \Psi(n+1, 2)\frac{\varphi_1}{1+||\varphi_1||^2}\epsilon(2) - \Psi(n+1,1)\frac{\varphi_0}{1+||\varphi_0||^2}\epsilon(1),
\end{aligned}$$

and hence,

$$\begin{aligned}
||\tilde{\theta}_n|| \leqslant& ||\Psi(n,0)||||\tilde{\theta}_0|| + ||\epsilon(n)|| + ||\Psi(n, n-1)||||\epsilon(n-1)|| \\
& + \cdots + ||\Psi(n,1)||||\epsilon(1)||.
\end{aligned} \tag{G1}$$

Noticing

$$\tau_n = n(h + mp) + 1,$$

by (16) we get $||\varphi_{\tau_n}|| = O(\tau_n^v)$. This together with the definition of M_i results in

$$M_i^2 = O\left(\tau_i^{4v}\right) = O\left(i^{4v}\right).$$

So, from (16) and Lemma 6 there exists $c_3 > 0$ such that

$$||\Psi(\tau_n, 0)|| \leqslant \exp\left(-c_3 \sum_{i=1}^{n} \frac{1}{i^{4v}}\right) = O\left(\exp\left(-c_4 (n+1)^{1-4v}\right)\right), \tag{G2}$$

where $c_4 = \frac{c_3}{1-4v} > 0$.

For any n, there exists k_n such that

$$\tau_{k_n} \leqslant n \leqslant \tau_{k_n + 1},$$

or

$$k_n(h + mp) + 1 \leqslant n \leqslant (k_n + 1)(h + mp) + 1.$$

So,

$$k_n + 1 \geqslant \frac{n-1}{h+mp}.$$

By (G2) we have

$$||\Psi(n,0)|| \leqslant ||\Psi(\tau_{k_n},0)|| = O\left(\exp\left(-c_5(k_n+1)^{1-4v}\right)\right) = O\left(\exp\left(-\alpha n^{1-4v}\right)\right), \tag{G3}$$

where $c_5 > 0$, $\alpha > 0$.

For $\Psi(n,k)$, by Lemma 5, we have

$$||\Psi(\tau_n,\tau_{n-1})|| \leqslant \left(1 - c_2' \frac{\delta^2}{M_n^2}\right)^{1/2}.$$

For any $1 \leqslant k \leqslant n$, by the definition of τ_n, there exists m such that $\tau_m \geqslant k$. So,

$$||\Psi(\tau_n,k)|| \leqslant ||\prod_{i=m+1}^{n} \Psi(\tau_i,\tau_{i-1})|| \leqslant \left(\prod_{i=m+1}^{n}\left(1 - c_2'\frac{\delta^2}{M_i^2}\right)\right)^{1/2}.$$

From (16) and Lemma 6 there exists $c_6 > 0$ such that

$$\begin{aligned} ||\Psi(\tau_n,k)|| &\leqslant \exp\left(-c_6 \sum_{i=m+1}^{n} \frac{1}{i^{4v}}\right) \\ &= O\left(\exp\left(-c_7(n+1)^{1-4v}\right)\right), \end{aligned} \tag{G4}$$

where $c_7 > 0$.

Hence, by (G2) and (G4) we can get

$$||\Psi(n,k)|| \leqslant ||\Psi(\tau_{k_n},k)|| = O\left(\exp\left(-c_8(k_n+1)^{1-4v}\right)\right) = O\left(\exp\left(-\beta n^{1-4v}\right)\right), \tag{G5}$$

where $c_8 > 0$, $\beta > 0$.

Therefore,

$$\lim_{n\to\infty} ||\Psi(n,1)|| + \cdots + ||\Psi(n,n)|| = O(1),$$

which together with (C1), (G1), (G3) and (G5), implies

$$||\tilde{\theta}_n|| = O(\varepsilon), \text{ as } n \to \infty.$$

Appendix H Simulation

Example 1. Tracking control with quantiezd outputs

Consider a system

$$A(z)y(k) = B(z)u(k-1) + C(z)w(k), \ k = 1, 2, ...$$

with

$$A(z) = \begin{bmatrix} 1+\frac{1}{2}z & 0 \\ 0 & 1+\frac{1}{3}z \end{bmatrix}, \quad B(z) = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \quad C(z) = \begin{bmatrix} 1+\frac{1}{2}z & \frac{1}{2}z \\ \frac{1}{3}z & 1+\frac{1}{3}z \end{bmatrix},$$

$w(k)$ being a 2-dimensional standard normal noise, the output $y(k)$ measured by (5) with $\varepsilon = 0.3$, and $y^*(k) = [1,1]^T$. Then, under the tracking control (8), the tracking error is shown in H1.

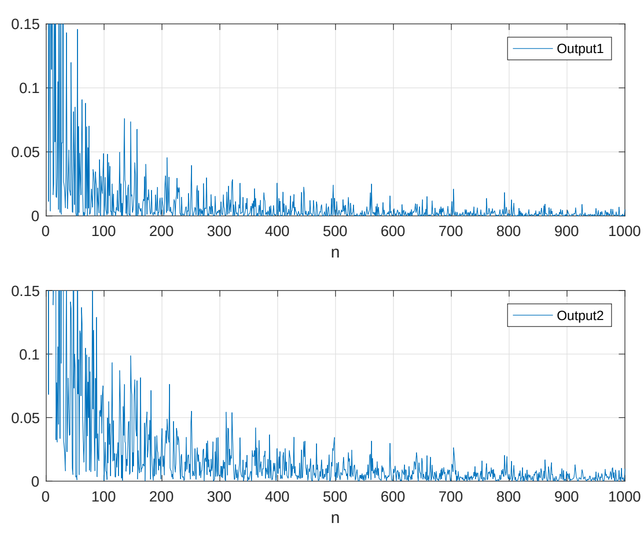

Figure H1 Trajectory of $\frac{1}{n}\sum_{k=1}^{n}||y_1(k) - y_1^*(k)||^2$ and $\frac{1}{n}\sum_{k=1}^{n}||y_1(k) - y_1^*(k)||^2$

Example 2. Parameter identification with quantiezd outputs

Consider a system

$$y(k) = ay(k-1) + bu(k-1),\ k = 1, 2, ...$$

with $\theta = [a, b]^T = [-1, 1]^T$ to be identified. The output $y(k)$ is measured by (5) with $\varepsilon = 0.01$. The projection algorithm (13) is used with initial $\theta_0 = [0, 0]^T$ and the control $u(k)$=-3, -1, 1, -3, -1, 1, -3 ... , k=1, 2, ... , and the $||\tilde{\theta}_n||$ is shown in H2.

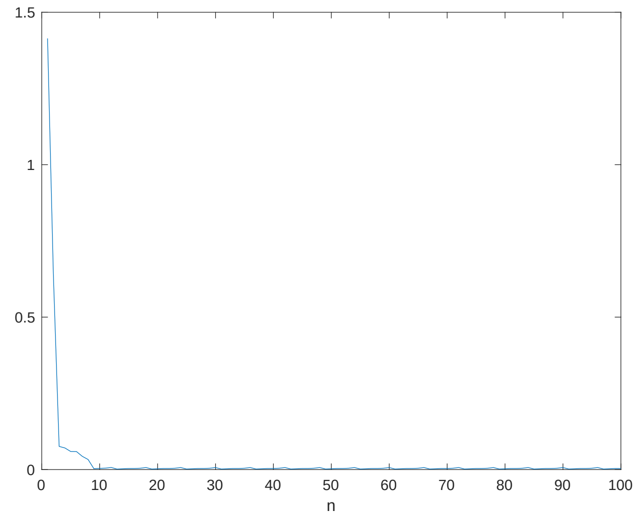

Figure H2 Trajectory of $||\tilde{\theta}_n||$

References

1 Chen H F, Guo L. Adaptive control via consistent estimation for deterministic systems. Int. J. Control, 1987, 45: 2183-2202